SOFR and €STR Discounting: Forward Convexity from Discount-Rate Transition

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ABSTRACT

Due the the upcoming change in price alignment interest that is anticipated for SOFR and €STR, a new definition of the ”hybrid” risk-neutral measure is developed, which is based on continuous reinvestment at the price alignment interest rate. A corresponding T-forward measure is defined, which is consistent with the treatment of cashflow obligations including bond prices and terminal payouts. Values obtained under the ”hybrid” measure remain unchanged when combined with explicit modeling of the compensation payout at the transition time, in addition to describing any MTM change for uncompensated positions. This new measure also allows continuous construction of the forward curve in a way that is convexity-free up to and beyond the discounting transition, whereas the original pricing measure will see a sudden impact on the forward curve at the transition point. Lastly the ”hybrid” measure provides a framework to model the forward curve convexity for trades with collateral agreements that do not transition simultaneously across different netting sets.

1 Introduction

There is a current global effort to prepare the financial industry for an upcoming cessation of the Libor benchmark interest rate, possibly coming as soon as January, 2022. There is now a concurrent task to encourage the increase of trading volume and activity linked to alternative reference rates (ARR) so that they are widely adopted and can successfully replace Libor in the coming days.

In the case of US dollars and associated USD Libor, there is now widespread acceptance that the Secured Overnight Financing Rate (SOFR), a daily aggregate of repo market transactions, will be the preferred ARR to replace USD Libor. The SOFR benchmark is relatively new, having only been published since April 2018. In past years the primary overnight rate in US dollars has been the effective federal funds rate (EFFR), which has been around for many years prior to SOFR, and is widely used in the USD overnight interest-rate swap (OIS) market as a floating rate benchmark.

The availability of OIS swap rates from this liquid market has facilitated the construction of risk-free discount curves. Ever since the widespread move to the OIS discounting methodology after the economic turmoil of 2008-2009, it is standard practice to discount US dollar cashflow obligations with the EFFR-based OIS discount curve. This is due to a close connection between the EFFR rate used in price alignment interest (PAI) for variation margin posted to central counterparty exchanges and the theoretical risk-free discount rate for cashflow obligations [1]. However, in accordance with the planned transition from Libor to SOFR, it is believed that updating the PAI and discounting curve for cleared USD swaps from EFFR to SOFR will enhance liquidity for SOFR. This has been proposed as a single-day transition to SOFR discounting, tentatively scheduled to occur on October 17, 2020 at both the LCH and CME exchanges.

A similar situation exists for the EONIA and €STR benchmark rates in the European Union. The recent start of publication for €STR on October 2, 2019 corresponds with a redefinition of EONIA to be €STR plus 8.5bps. Both LCH and Eurex have announced similar decisions to shift their discounting methodology to be based on the €STR benchmark by October 2020. Common throughout these proposals is the recognition that there will be an impact on the mark-to-market value (MTM) of open positions that are currently discounted at EFFR or EONIA.

In order to mitigate the impact of the discounting change on the MTM of current positions, a compensation mechanism has been devised that will provide cash compensation for the MTM impact at the same time as the official change of the PAI benchmark rate. This cash compensation has been included with all current proposals and can be understood as a single future cashflow that is stochastic and exactly equal to a future change in MTM that will occur for each individual position held with the exchange that is subject to discounting in the MTM calculation.

Risk-free discounting has a central role in the determination of present value of risky assets, within the risk-neutral measure as defined by the fundamental theory of asset pricing. It is therefore important to consider how the mathematical framework for arbitrage-free derivative pricing is impacted by these proposed changes to discounting rates in the near future. The purpose of the following sections is to evaluate the impact on discount curve construction and forward curves in the pricing of fully collataralized derivatives, which are subject to changes in the benchmark rate used for the rate of interest earned on their variation margin at predictable times in the future.

2 Hybrid EFFR/SOFR Bank Account Measure

There is a well-known connection between the discount rate and collateral rate on theoretical grounds, which can be intuitively understood as a result of arbitrage-free replication of a risk-free hedged position that is subject to the requirement of full cash collateralization [1]. Given stochastic interest-rates, the discount factor is generally defined by the change of measure from the risk-neutral to T-forward measure corresponding to a zero-coupon bond (ZCB) numeraire with terminal cashflow at time T.

To account for the upcoming PAI change from EFFR to SOFR, this can be understood as a single collateral account that accrues at the EFFR rate up until the time of the switch, denoted T^*. This updates our definition of the bank account numeraire for risk-neutral measure to be:

$$B(t) = \begin{cases} e^{\int_t^T r_A(u)du} & t \leq T \leq T^* \\ e^{\int_t^{T^*} r_A(u)du + \int_{T^*}^T r_B(u)du} & t \leq T^* \leq T \\ e^{\int_t^T r_B(u)du} & T^* \leq t \leq T \end{cases} \quad (1)$$

In Eq. (1), the pair (r_A, r_B) represents either the pair (EFFR,SOFR) or (EONIA,€STR) in the USD and EUR currencies, respectively. The impact of the hybrid PAI methodology is theoretically equivalent to a dynamic reinvestment at the stochastic overnight rate $r_A(t)$ until T^*, then changing to the reinvestment at rate $r_B(t)$. This is a practical hedging strategy accessible to investors, and replicates the collateral rate including the change at T^*.

It is illustrative to first consider the case of deterministic interest rates, where the following expression is obtained for the ZCB price that is equivalent to the present value of the continuous reinvestment strategy.

$$P(t,T) = e^{-\int_t^{T^*} r_A(u)du - \int_{T^*}^T r_B(u)du} = \frac{e^{-\int_t^{T^*} r_A(u)du - \int_t^T r_B(u)du}}{e^{-\int_t^{T^*} r_B(u)du}} = \frac{P_A(t,T^*)P_B(t,T)}{P_B(t,T^*)} \quad (2)$$

Note that $P_A(t,T)$ is based entirely on the overnight rate $r_A(t)$ and similarly $P_B(t,T)$ is based solely on $r_B(t)$. In the case of deterministic interest rates, the expected value of continuous reinvestment is equivalent to the discounted present value of a unit cashflow obligation at the terminal time T, which is readily apparent because of the exact equivalence of the ZCB and the reinvestment strategy. Rather than actually requiring deterministic interest rates, we seek a probability measure which can recover this result for the case of stochastic interest rates, similar to the original T-forward measure, except accounting for the change in collateral rate. It will now be demonstrated that we in fact seek the following definition for a "hybrid" ZCB:

$$P_H(t,T) = \begin{cases} P_A(t,T) & t \le T \le T^* \\ P_A(t,T^*)\dfrac{P_B(t,T)}{P_B(t,T^*)} & t \le T^* \le T \\ P_B(t,T) & T^* \le t \le T \end{cases} \tag{3}$$

In the case $t \le T^* \le T$, the "hybrid" ZCB $P_H(t,T)$ can be recognized as the value of a time t investment in $P_A(t,T)$ units of a forward ZCB $P_B(T^*,T)$. For the case $t \le T \le T^*$ it is just the regular ZCB based on $r_A(t)$, and similarly for $T^* \le t \le T$ it is the ZCB based solely on $r_B(t)$. Using $P_H(t,T)$ as the numeraire asset leads the following Radon–Nikodym derivative:

$$\frac{dQ}{dQ^T}(t,\tau) = \begin{cases} \dfrac{P_A(t,T)}{P_A(\tau,T)}\dfrac{B(\tau)}{B(t)} & t \le T \le T^* \\ \dfrac{P_A(t,T^*)P_B(t,T)}{P_B(t,T^*)P_B(\tau,T)}\dfrac{B(\tau)}{B(t)} & t \le T^* \le T \\ \dfrac{P_B(t,T)}{P_B(\tau,T)}\dfrac{B(\tau)}{B(t)} & T^* \le t \le T \end{cases} \tag{4}$$

It is apparent that both the hybrid ZCB $P_H(t,T)$ and its corresponding T-forward measure are identical to those for $P_A(t,T)$ prior to the discounting transition, and also identical to $P_B(t,T)$ after the transition has occured. The fundamentally new result is for application specifically in the situation where the transition has yet to occur and the terminal payout time is after the transtion point, i.e. when $t \le T^* \le T$. In this situation we can calculate the present value of a derivative payout $X(T)$ in the "hybrid" T-forward measure as:

$$\mathbb{E}^Q\left[\frac{B(t)}{B(T)}X(T)\middle|\mathscr{F}_t\right] = \mathbb{E}^{Q^T}\left[\frac{dQ}{dQ^T}(t,T)\frac{B(t)}{B(T)}X(T)\middle|\mathscr{F}_t\right] = \frac{P_A(t,T^*)P_B(t,T)}{P_B(t,T^*)}\mathbb{E}^{Q^T}\left[X(T)|\mathscr{F}_t\right] \tag{5}$$

The result of Eq. (5) recovers the deterministic discount factor in Eq. (2), however without the assumption of deterministic interest rates. This is achieved by the definition of a new probability measure, which is equivalent to the original risk-neutral measure only incorporating the change in the collateral interest rate via a numeraire asset that is a dynamically replicable position in a B-rate ZCB starting at T^*, discounted by the value of a spot-starting A-rate ZCB. Note that the time t price of the forward ZCB is consistently defined as the numeraire asset under this same measure:

$$\mathbb{E}^Q\left[\frac{B(t)}{B(T^*)}P_B(T^*,T)\middle|\mathscr{F}_t\right] = P_A(t,T^*)\,\mathbb{E}^{Q^{T^*}}\left[\frac{P_B(T^*,T)}{P_B(T^*,T^*)}\middle|\mathscr{F}_t\right] = P_A(t,T^*)\frac{P_B(t,T)}{P_B(t,T^*)} \tag{6}$$

It has been shown that this is a convenient measure change to recover the deterministic discount factor under the relaxed assumption of stochastic interest rates. In the next section it will be shown that there is an important use of this measure in application to the modeling of the MTM compensation at the central counterparty PAI methodology transition point.

3 Equivalence to the Perfect Compensation of Value Transfer

The MTM impact of the PAI and discount curve transition is expected to be balanced with a cash compensation at the time of the transition. The size of the cash compensation is entirely based on the future realized impact of the collateral rate change. For a terminal payout $X(T)$, where $T^* \le T$, the size of the cash compensation at T^* is:

$$\begin{aligned} G(T^*) &= P_A(T^*,T)\mathbb{E}^{Q^{T_A}}\left[X(T)|\mathscr{F}_{T^*}\right] - P_B(T^*,T)\mathbb{E}^{Q^{T_B}}\left[X(T)|\mathscr{F}_{T^*}\right] \\ &= \mathbb{E}^{Q_A}\left[\frac{B_A(T^*)}{B_A(T)}X(T)\middle|\mathscr{F}_{T^*}\right] - \mathbb{E}^{Q}\left[\frac{B(T^*)}{B(T)}X(T)\middle|\mathscr{F}_{T^*}\right] \end{aligned} \tag{7}$$

Note this expression involves two separate probability measures, since the compensation is in fact defined by the difference in valuation under these two measures. This of course accounts for the separate discounting curves being used before and after the PAI transition, but notably also includes any secondary impacts from the forward curve being impacted. The differences in forward curves will be discussed in detail in the next section.

The present value of the compensation at time t is obtained from the following:

$$\begin{aligned} G(t) &= \mathbb{E}^{Q_A}\left[\frac{B_A(t)}{B_A(T^*)}\mathbb{E}^{Q_A}\left[\frac{B_A(T^*)}{B_A(T)}X(T)\middle|\mathscr{F}_{T^*}\right]\middle|\mathscr{F}_t\right] - \mathbb{E}^{Q}\left[\frac{B(t)}{B(T^*)}\mathbb{E}^{Q}\left[\frac{B(T^*)}{B(T)}X(T)\middle|\mathscr{F}_{T^*}\right]\middle|\mathscr{F}_t\right] \\ &= P_A(t,T)\,\mathbb{E}^{Q^{T_A}}\left[X(T)|\mathscr{F}_t\right] - P_A(t,T^*)\frac{P_B(t,T)}{P_B(t,T^*)}\mathbb{E}^{Q^T}\left[X(T)|\mathscr{F}_t\right] \end{aligned} \tag{8}$$

The first term in Eq. (8) represents the present value of the position without any modification for the change in PAI interest rate, while the second term reflects the true present value as given by the hybrid measure. This leads to:

$$\mathbb{E}^{Q}\left[\frac{B(t)}{B(T)}X(T)\middle|\mathscr{F}_t\right] + G(t) = \mathbb{E}^{Q_A}\left[\frac{B_A(t)}{B_A(T)}X(T)\middle|\mathscr{F}_t\right] \tag{9}$$

This result shows that the present value of any position, calculated solely under the discount rate $r_A(t)$, is exactly equivalent to value obtained for the position discounted under the hybrid measure defined in the previous section, plus an explicit value-neutral compensation at the transition time T^*. This demonstrates how the hybrid measure self-consistently models the effect of the PAI change by replicating the perfect compensation for the original $r_A(T)$ discounting measure.There are also wider uses of the hybrid measure beyond the scope of the discussion thus far, such as in the case of a future PAI transition without any compensation, so that $G(t) = G(T^*) = 0$. This would break the equality in Eq. (9) and generally show a MTM impact in this situation, which can be immediately realized by using the hybrid measure.

4 Impact on Forward Curve Construction

It is clear that pricing within the hybrid measure will require a new construction for the discount curve, which combines the discount factors from the two curves separately based on $r_A(t)$ and $r_B(t)$, according to Eq. (3). However there is the question of how to separately source the $P_A(t,T)$ and $P_B(t,T)$ discount curves that are to be combined in Eq. (3). In the situation where $t < T^*$ we can assume the availability of OIS and futures instruments based on $r_A(t)$ that are discounted according to $r_A(t)$ PAI, which can be used to construct the discount curve $P_A(t,T)$ at time t. Similarly we can assume there are market data quotes for OIS and futures instruments based on $r_B(t)$ that are discounted according to $r_A(t)$ PAI. Given the initial construction of $P_A(t,T)$, the discount curve $P_B(t,T)$ can be implied from these instruments, and then the hybrid curve $P_H(t,T)$ in Eq. (3) can be built up to T^*. After T^*, the remainder of $P_B(t,T)$ can be calibrated from instruments based on $r_B(t)$, and in the last step the long-end of $P_A(t,T)$ can be built from instruments based on $r_A(t)$ collateralized by $r_B(t)$. Unless the maturities of the instruments align perfectly with T^*, some extrapolation of $P_A(t,T)$ may be needed to handle the first instrument that spans the PAI transition point.

Provided that the instruments used for discount curve calibration are quoted at par, there is no impact on their value from the use of $P_H(t,T)$ for discounting, but the main impact will be on the implied forward curve obtained in the curve calibration procedure. This leads to an important consideration about the use of the forward in pricing. This can be shown by a derivation of the original forward curve based on $r_A(t)$ in the hybrid measure:

$$\begin{aligned} F_A(t,T) &= \frac{\mathbb{E}^{Q}\left[e^{-\int_t^T r_A(u)du}X(T)\middle|\mathscr{F}_t\right]}{P_A(t,T)} = \frac{\mathbb{E}^{Q}\left[e^{-\int_t^{T^*} r_A(u)du-\int_{T^*}^{T} r_B(u)du}e^{-\int_{T^*}^{T}(r_A(u)-r_B(u))du}X(T)\middle|\mathscr{F}_t\right]}{P_A(t,T)} \\ &= \frac{P_H(t,T)}{P_A(t,T)}\mathbb{E}^{T}\left[e^{-\int_{T^*}^{T}(r_A(u)-r_B(u))du}X(T)\middle|\mathscr{F}_t\right] \end{aligned} \tag{10}$$

The result of Eq. (10) shows that the original $r_A(t)$ based forward curve $F_A(t,T)$ is model-dependent in the hybrid measure, and requires specification of the dynamics of the spread $(r_A(t) - r_B(t))$ in the hybrid measure in order to calculate. Switching to the hybrid measure will have an immediate impact on the forward curve obtained through curve construction, essentially involving a convexity adjustment for the original forward curve after the PAI transition. Eq. (10) also demonstrates that if separate collateral agreements are held for trades that do not involve a PAI transition, then these trades will require convexity adjustments in order to price as soon as the PAI transition is known for instruments involved in curve construction.

5 Conclusion

The hybrid measure introduced here is shown to be fully self-consistent with the treatment of cashflow obligations including ZCB prices and terminal payouts equivalently under risk neutral or T-forward expectations. The benefit of having a measure which accounts for the PAI transition is that it remains value neutral when combined with explicit modeling of the compensation payout at the transition time, in addition to providing a way to incorporate the MTM change for current positions for the case where there is no compensation provided for the PAI transition impact. The hybrid measure also allows continuous construction of the forward curve in a way that is convexity-free up to and beyond the PAI transition, whereas the original pricing measure does not take this into account and will have a sudden impact on the forward curve at the transition point. Finally, the hybrid measure allows further analysis of the forward curve convexity for trades with collateral agreements that do not have PAI transitions, or in the case where the PAI transition is not simultaneous across different netting sets.

References

1. Piterbarg, V. Funding beyond discounting: collateral agreements and derivatives pricing. *Risk* **23**, 97 (2010).